

# Vogel's Approximation Method

## Algorithm

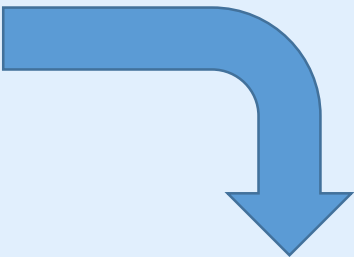
### **Vogel's Approximation Method (VAM) or penalty method**

This method is preferred over the NWCM and LCM, because the initial basic feasible solution obtained by this method is either optimal solution or very nearer to the optimal solution.

### **Vogel's Approximation Method (VAM) Steps (Rule)**

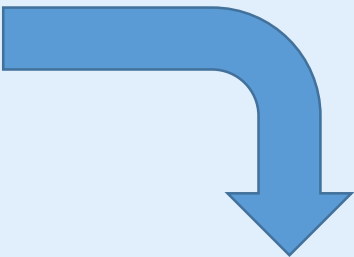
|         |                                                                                                                                                                                                                                                                         |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Step-1: | Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.                                                                                                              |
| Step-2: | Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.                                                                                                   |
| Step-3: | Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell.<br>If there is a tie in the values of penalties then select the cell where maximum allocation can be possible |
| Step-4: | Adjust the supply & demand and cross out (strike out) the satisfied row or column.                                                                                                                                                                                      |
| Step-5: | Repeat this steps until all supply and demand values are 0.                                                                                                                                                                                                             |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



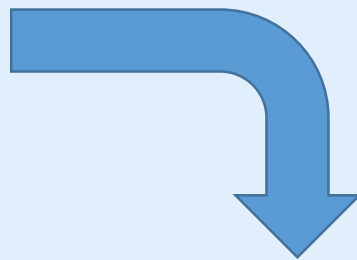
|                | $D_1$          | $D_2$         | $D_3$          | $D_4$          | Supply | Row Penalty    |
|----------------|----------------|---------------|----------------|----------------|--------|----------------|
| $S_1$          | 19             | 30            | 50             | 10             | 7      | $9 = 19 - 10$  |
| $S_2$          | 70             | 30            | 40             | 60             | 9      | $10 = 40 - 30$ |
| $S_3$          | 40             | 8             | 70             | 20             | 18     | $12 = 20 - 8$  |
| Demand         | 5              | 8             | 7              | 14             |        |                |
| Column Penalty | $21 = 40 - 19$ | $22 = 30 - 8$ | $10 = 50 - 40$ | $10 = 20 - 10$ |        |                |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



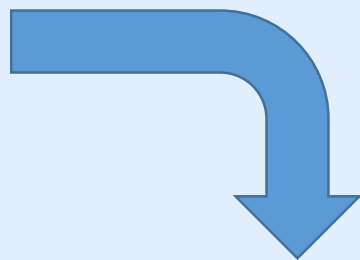
|                | $D_1$          | $D_2$         | $D_3$          | $D_4$          | Supply | Row Penalty    |
|----------------|----------------|---------------|----------------|----------------|--------|----------------|
| $S_1$          | 19             | 30            | 50             | 10             | 7      | $9 = 19 - 10$  |
| $S_2$          | 70             | 30            | 40             | 60             | 9      | $10 = 40 - 30$ |
| $S_3$          | 40             | 8             | 70             | 20             | 18     | $12 = 20 - 8$  |
| Demand         | 5              | 8             | 7              | 14             |        |                |
| Column Penalty | $21 = 40 - 19$ | $22 = 30 - 8$ | $10 = 50 - 40$ | $10 = 20 - 10$ |        |                |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



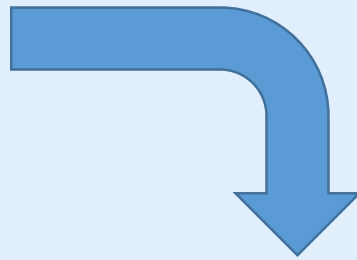
|                | $D_1$          | $D_2$ | $D_3$          | $D_4$          | Supply | Row Penalty    |
|----------------|----------------|-------|----------------|----------------|--------|----------------|
| $S_1$          | 19             | 30    | 50             | 10             | 7      | $9 = 19 - 10$  |
| $S_2$          | 70             | 30    | 40             | 60             | 9      | $20 = 60 - 40$ |
| $S_3$          | 40             | 8(8)  | 70             | 20             | 10     | $20 = 40 - 20$ |
| Demand         | 5              | 0     | 7              | 14             |        |                |
| Column Penalty | $21 = 40 - 19$ | --    | $10 = 50 - 40$ | $10 = 20 - 10$ |        |                |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



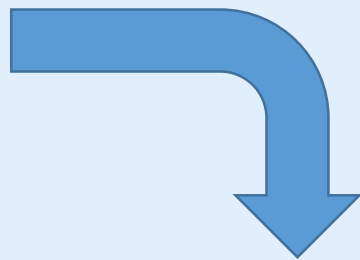
|                | $D_1$ | $D_2$ | $D_3$          | $D_4$          | Supply | Row Penalty    |
|----------------|-------|-------|----------------|----------------|--------|----------------|
| $S_1$          | 19(5) | 30    | 50             | 10             | 2      | $40 = 50 - 10$ |
| $S_2$          | 70    | 30    | 40             | 60             | 9      | $20 = 60 - 40$ |
| $S_3$          | 40    | 8(8)  | 70             | 20             | 10     | $50 = 70 - 20$ |
| Demand         | 0     | 0     | 7              | 14             |        |                |
| Column Penalty | --    | --    | $10 = 50 - 40$ | $10 = 20 - 10$ |        |                |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



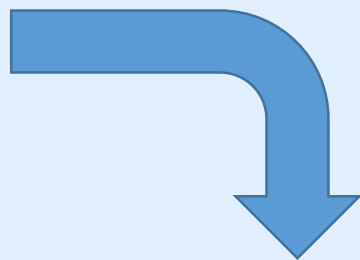
|                | $D_1$ | $D_2$ | $D_3$          | $D_4$          | Supply | Row Penalty    |
|----------------|-------|-------|----------------|----------------|--------|----------------|
| $S_1$          | 19(5) | 30    | 50             | 10             | 2      | $40 = 50 - 10$ |
| $S_2$          | 70    | 30    | 40             | 60             | 9      | $20 = 60 - 40$ |
| $S_3$          | 40    | 8(8)  | 70             | 20(10)         | 0      | --             |
| Demand         | 0     | 0     | 7              | 4              |        |                |
| Column Penalty | --    | --    | $10 = 50 - 40$ | $50 = 60 - 10$ |        |                |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



|                | $D_1$ | $D_2$ | $D_3$ | $D_4$  | Supply | Row Penalty    |
|----------------|-------|-------|-------|--------|--------|----------------|
| $S_1$          | 19(5) | 30    | 50    | 10(2)  | 0      | --             |
| $S_2$          | 70    | 30    | 40    | 60     | 9      | $20 = 60 - 40$ |
| $S_3$          | 40    | 8(8)  | 70    | 20(10) | 0      | --             |
| Demand         | 0     | 0     | 7     | 2      |        |                |
| Column Penalty | --    | --    | 40    | 60     |        |                |

|        | D1 | D2 | D3 | D4 | Supply |
|--------|----|----|----|----|--------|
| S1     | 19 | 30 | 50 | 10 | 7      |
| S2     | 70 | 30 | 40 | 60 | 9      |
| S3     | 40 | 8  | 70 | 20 | 18     |
| Demand | 5  | 8  | 7  | 14 |        |



|                | $D_1$            | $D_2$           | $D_3$         | $D_4$             | Supply | Row Penalty |
|----------------|------------------|-----------------|---------------|-------------------|--------|-------------|
| $S_1$          | <del>19(5)</del> | <del>30</del>   | <del>50</del> | <del>10(2)</del>  | 0      | --          |
| $S_2$          | 70               | 30              | 40            | <del>60(2)</del>  | 7      | 40          |
| $S_3$          | <del>40</del>    | <del>8(8)</del> | <del>70</del> | <del>20(10)</del> | 0      | --          |
| Demand         | 0                | 0               | 7             | 0                 |        |             |
| Column Penalty | --               | --              | 40            | --                |        |             |

|                   | $D_1$ | $D_2$ | $D_3$ | $D_4$  | Supply | Row Penalty                 |
|-------------------|-------|-------|-------|--------|--------|-----------------------------|
| $S_1$             | 19(5) | 30    | 50    | 10(2)  | 7      | 9   9   40   40   --   --   |
| $S_2$             | 70    | 30    | 40(7) | 60(2)  | 9      | 10   20   20   20   20   40 |
| $S_3$             | 40    | 8(8)  | 70    | 20(10) | 18     | 12   20   50   --   --   -- |
| Demand            | 5     | 8     | 7     | 14     |        |                             |
| Column<br>Penalty | 21    | 22    | 10    | 10     |        |                             |
|                   | 21    | --    | 10    | 10     |        |                             |
|                   | --    | --    | 10    | 10     |        |                             |
|                   | --    | --    | 10    | 50     |        |                             |
|                   | --    | --    | 40    | 60     |        |                             |
|                   | --    | --    | 40    | --     |        |                             |

The minimum total transportation cost =  $19 \times 5 + 10 \times 2 + 40 \times 7 + 60 \times 2 + 8 \times 8 + 20 \times 10 = 779$